Kittel

3.5 (a) The nearest-neighbor repulsion being AR^{-n} implies

$$U_{tot} = N\left(\frac{2A}{R^n} - \frac{\alpha q^2}{R}\right)$$

$$\left.\frac{dU_{tot}}{dR}\right|_{R_0} = 0 = (-n)2AR_0^{-n-1} + \alpha q^2 R_0^{-2}$$

$$\Rightarrow \boxed{\alpha q^2 = 2nAR_0^{-n+1}}$$

Thus $2AR^{-n} = \dfrac{\alpha q^2 R^{-1}}{n}$,

$$U_{tot} = N\left(\frac{\alpha q^2 R^{-1}}{n} - \alpha q^2 R^{-1}\right)$$

$$= \frac{N\alpha q^2}{R_0}\left[\frac{1}{n} - 1\right], \text{ using } \alpha = 2\ln 2,$$

$$\boxed{U_{tot} = \frac{-2Nq^2\ln 2}{R_0}\left[1 - \frac{1}{n}\right].}$$

(b) $$T = -\frac{dU}{dR} = N\left(2AnR^{-n-1} - \alpha q^2 R^{-2}\right)\Big|_{R=R_0(1-\delta)}.$$

$$R^{-n-1} = R_0^{-n-1}(1-\delta)^{-n-1}, \quad R^{-2} = R_0^{-2}(1-\delta)^{-2},$$

expanding in δ gives.

$$R^{-n-1} \approx R_0^{-n-1}(n+1)\delta,$$

$$R^{-2} \approx R_0^{-2}(2)\delta,$$

$$T = N\left(2AnR_0^{-n-1}(n+1)\delta - \alpha q^2 R_0^{-2}(2)\delta\right].$$

$$= N\left[2An R_0^{-n-1}(n+1) - \alpha q^2 R_0^{-2}(2)\right]\delta.$$

$$\delta W = N\left[2An R_0^{-n-1}(n+1) - \alpha q^2 R_0^{-2}(2)\right]\delta^2(-R_0)$$

$$= N\left[\alpha q^2 R_0^{-1}(2) - \alpha q^2 (R_0^{-1})(n+1)\right]\delta^2$$

$$= (-1)\frac{N\alpha q^2}{R_0}[n-1]\,\delta^2$$

dividing by $2N$ to get work done per particle:

$$\frac{\cancel{N}\alpha q^2}{2R_0}[n-1]\delta^2 = \boxed{\frac{\cancel{\alpha}\ \ln 2\, q^2}{R_0}[n-1]\delta^2.}$$